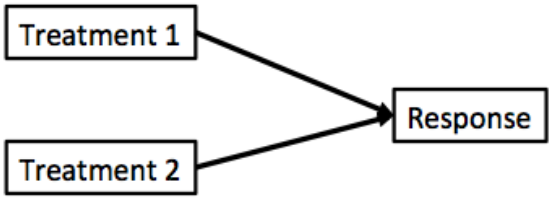
# Multiple Predictor Variables: Regression & the General Linear Model

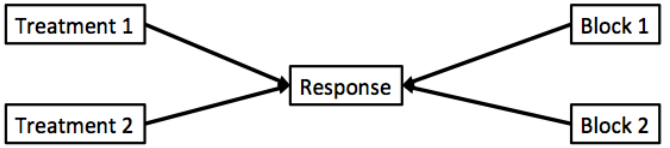
## Contrasts for a Multiway ANOVA

```
library(contrast)
contrast(zoop_lm,
         list(treatment="low", block=levels(zoop$block)),
         list(treatment="high", block=levels(zoop$block)),
         type="average")

# lm model parameter contrast
#
#   Contrast   S.E.    Lower Upper    t df Pr(>|t|)
# 1     0.62 0.2895 -0.04755 1.288 2.14  8   0.0646
```

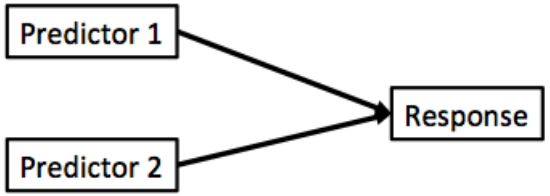
## One-Way ANOVA Graphically




## Two-Way ANOVA Graphically

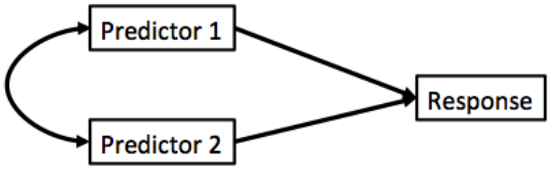


## Multiple Linear Regression?




Note no connection between predictors, as in ANOVA. This is ONLY true if we have manipulated it so that there is no relationship between the two.

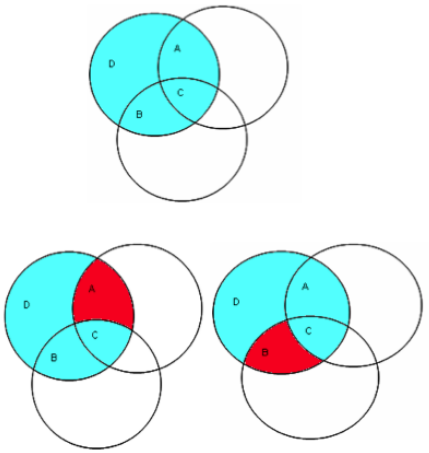
## Multiple Linear Regression




Curved double-headed arrow indicates COVARIANCE between predictors that we must account for.

## Semi-Partial Correlation

- Semi-Partial correlation asks how much of the variation in a response is due to a predictor after the contribution of other predictors has been removed
- How much would $R^2$ change if a variable was removed?
- A / (A+B+C+D)
- $sr_{y1} = \frac{r_{y1} - r_{y2}y_{12}}{\sqrt{1-r_{12}^2}}$




## Calculating Multiple Regression Coefficients with OLS

$$\boldsymbol{Y} = \boldsymbol{bX} + \boldsymbol{\epsilon}$$

Remember in Simple Linear Regression $b = \frac{cov_{xy}}{var_x}$?

In Multiple Linear Regression $\boldsymbol{b} = \boldsymbol{cov_{xy}} \boldsymbol{S_x^{-1}}$

where $\boldsymbol{cov_{xy}}$ is the covariances of $\boldsymbol{x_i}$ with $\boldsymbol{y}$ and $\boldsymbol{S_x^{-1}}$ is the variance/covariance matrix of all *Independent variables*

OR $bi = \frac{cov_{xy} - \sum cov_{x1xj} b_j}{var_{(x)}}$

## Calculating Multiple Regression Coefficients with OLS

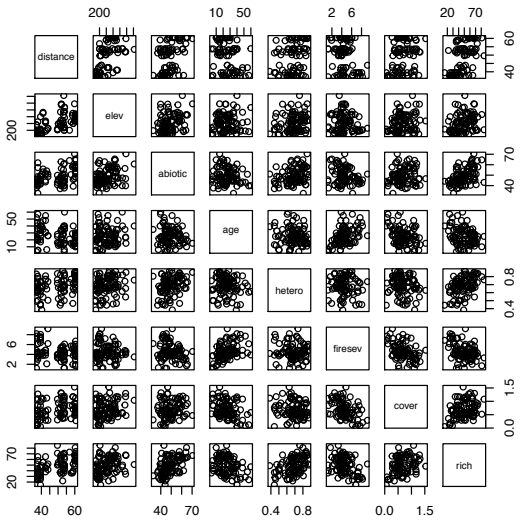
$$\boldsymbol{Y} = \boldsymbol{bX} + \boldsymbol{\epsilon}$$

Coefficient Estimates: $E[\hat{\boldsymbol{\beta}}] = \boldsymbol{cov_{xy}} \boldsymbol{S_x^{-1}}$

Coefficient Variance: $Var[\hat{\beta}_i] = \frac{\boldsymbol{\sigma^2}}{\boldsymbol{SXX_i}}$

Five year study of wildfires & recovery in Southern California shurblands in 1993. 90 plots (20 x 50m)
(data from Jon Keeley et al.)

## Many Things may Influence Species Richness




## Many Things may Influence Species Richness

```
klm <- lm(rich ~ cover + firesev + hetero, data=keeley)
```

## Checking for Multicollinearity: Correlation Matrices

```
with(keeley, cor(cbind(cover, firesev, hetero)))

#            cover  firesev   hetero
# cover    1.0000 -0.43713 -0.16838
# firesev -0.4371  1.00000 -0.05236
# hetero  -0.1684 -0.05236  1.00000
```

Correlations over 0.4 can be problematic, but, they may be OK even as high as 0.8. Beyond this, are you getting unique information from each variable?

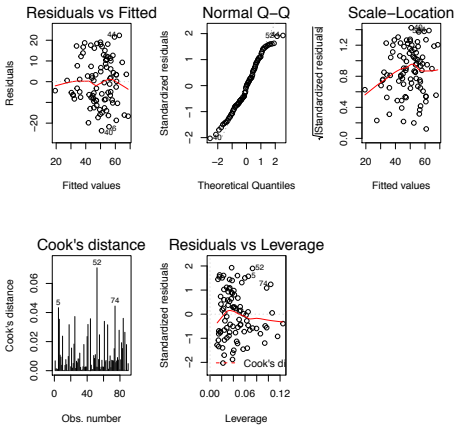
## Checking for Multicollinearity: Variance Inflation Factor

$$VIF = \frac{1}{1 - R_j^2}$$

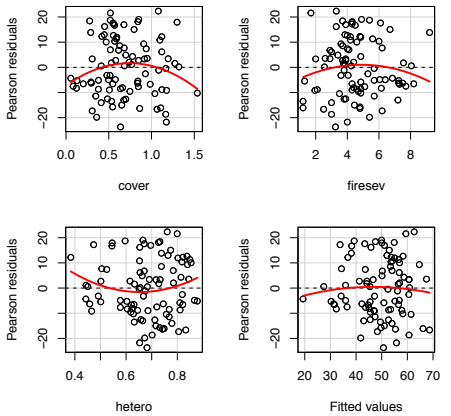
```
vif(klm)

#   cover firesev  hetero
#   1.295   1.262   1.050
```

VIF $>$ 5 or 10 can be problematic and indicate an unstable solution.

## Other Diagnostics as Usual!



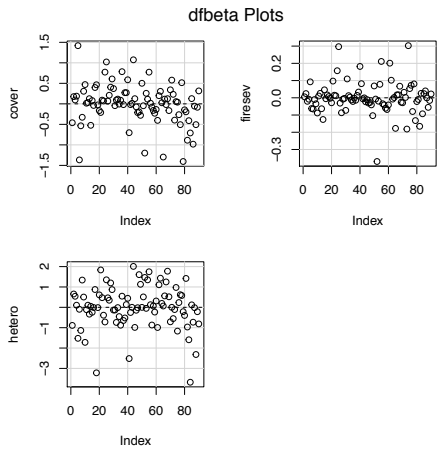

## Other Diagnostics as Usual!




```
#            Test stat Pr(>|t|)
# cover         -1.602    0.113
# firesev       -1.087    0.280
```

## New Diagnostic for Outliers: Leave One Out

```
dfbetaPlots(klm)
```




## Which Variables Explained Variation: Type II Marginal SS

```
Anova(klm)

# Anova Table (Type II tests)
#
# Response: rich
#           Sum Sq Df F value  Pr(>F)
# cover       1674  1   12.01 0.00083
# firesev      636  1    4.56 0.03554
# hetero      4865  1   34.91 6.8e-08
# Residuals  11985 86
```

If order of entry matters, can use type I. Remember, what models are you comparing?

## The coefficients

```
summary(klm)$coef

#             Estimate Std. Error t value  Pr(>|t|)
# (Intercept)    1.679    10.6737  0.1573 8.754e-01
# cover         15.558     4.4886  3.4661 8.264e-04
# firesev       -1.817     0.8506 -2.1357 3.554e-02
# hetero        65.992    11.1694  5.9082 6.757e-08

cat(paste("R^2 = ", round(summary(klm)$r.squared, 2), sep=""))

# R^2 = 0.41
```

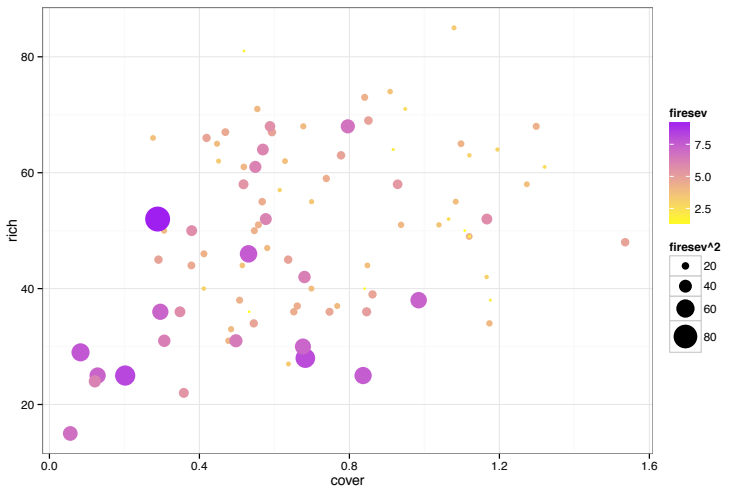
If order of entry matters, can use type I. Remember, what models are you comparing?

## Comparing Coefficients on the Same Scale

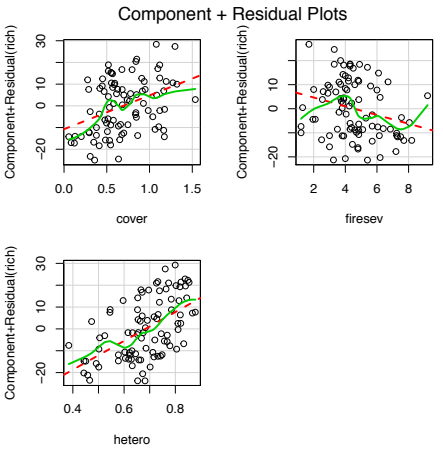
$$r_{xy} = b_{xy}\frac{sd_x}{sd_y}$$

```
library(QuantPsyc)
lm.beta(klm)

#   cover firesev  hetero
#  0.3267 -0.1987  0.5016
```

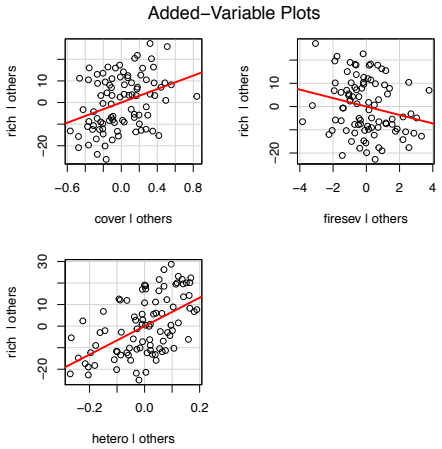
## Visualization of Multivariate Models is Difficult




## Component-Residual Plots Aid in Visualization



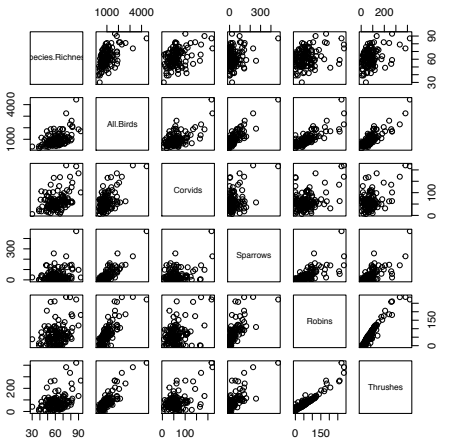

## Added Variable Plots for Unique Contribution of a Variable




Analagous to the A part of the three-circle diagram from earlier.

## Exercise: Bird Species Richness

- Which bird abundances influence Species Richness?
- Can we use every variable?
- Visualize Resuits



## All of the Birds!

```
wnv_lm_vif <- lm(Species.Richness ~ Corvids +
                                    Sparrows +
                                    Robins +
                                    Thrushes , data=wnv)
```

## Correlation Problems

```
cor(wnv[,c(3:8)])

#                   Species.Richness All.Birds Corvids
# Species.Richness            1.0000    0.5058  0.4326
# All.Birds                   0.5058    1.0000  0.5964
# Corvids                     0.4326    0.5964  1.0000
# Sparrows                    0.2406    0.8465  0.3846
# Robins                      0.2928    0.8075  0.4028
# Thrushes                    0.3859    0.8531  0.4960
#                   Sparrows Robins Thrushes
# Species.Richness    0.2406 0.2928   0.3859
# All.Birds           0.8465 0.8075   0.8531
# Corvids             0.3846 0.4028   0.4960
# Sparrows            1.0000 0.7083   0.7286
# Robins              0.7083 1.0000   0.9572
```

## Multicollinearity Problems

```
vif(wnv_lm_vif)

#  Corvids Sparrows   Robins Thrushes
#    1.449    2.145   13.050   15.060
```

## Odd Results from Robins and Sparrows

```
summary(wnv_lm_vif)

#
# Call:
# lm(formula = Species.Richness ~ Corvids + Sparrows + Robins +
#     Thrushes, data = wnv)
#
# Residuals:
#     Min      1Q  Median      3Q     Max
# -24.997  -6.250  -0.093   6.827  22.074
#
# Coefficients:
#             Estimate Std. Error t value Pr(>|t|)
# (Intercept)  53.3019     1.6681   31.95   <2e-16
# Corvids       0.0732     0.0262    2.79   0.0060
# Sparrows     -0.0150     0.0202   -0.74   0.4596
# Robins       -0.1235     0.0502   -2.46   0.0152
# Thrushes      0.1538     0.0471    3.27   0.0014
#
```

## A New Model

```
wnv_lm <- lm(Species.Richness ~ Corvids +
                                Sparrows +
                                Robins, data=wnv)
```

## No Multicollinearity Problem

```
vif(wnv_lm)

#  Corvids Sparrows   Robins
#    1.223    2.055    2.091
```

## A Corvid Story

```
Anova(wnv_lm)

# Anova Table (Type II tests)
#
# Response: Species.Richness
#           Sum Sq  Df F value  Pr(>F)
# Corvids     1793   1   18.36 3.6e-05
# Sparrows       1   1    0.01    0.94
# Robins       160   1    1.64    0.20
# Residuals  12306 126
```

## A Corvid Story

```
crPlots(wnv_lm)
```