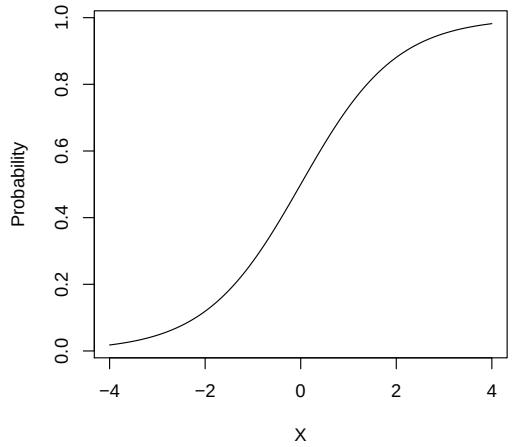
Logistic Regression

The Logitistic Curve for Probabilities



The Logitistic Function

$$p = \frac{e^{(a+bx)}}{1 + e^{(a+bx)}}$$

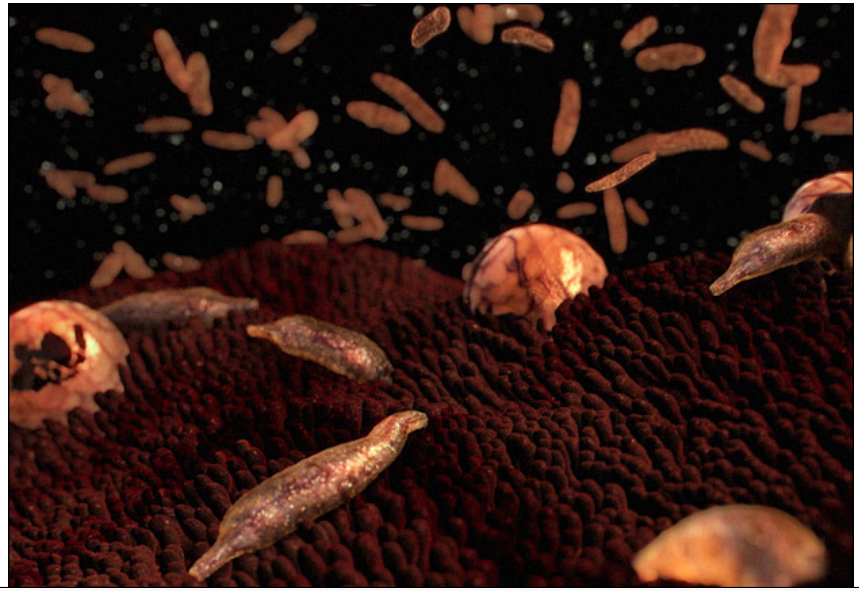
The Log Odds

$$a + bx = Log\frac{p}{1-p}$$

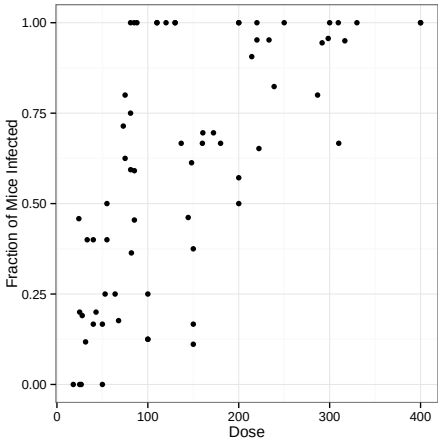
The Odds

$$Odds = \frac{p}{1-p}$$

Cryptosporidium



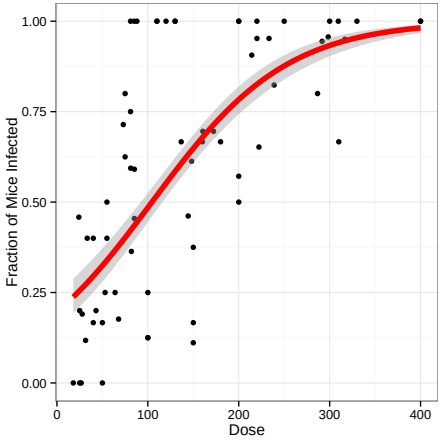
Fraction of Mice Infected = Probability of Infection



Two Different Ways of Writing the Model

```
# 1) using Heads, Tails
glm(cbind(Y, N-Y) ~ Dose, data=crypto, family=binomial)
#
#
# 2) using weights as size parameter for Binomial
glm(Y/N ~ Dose, weights=N, data=crypto, family=binomial)
```

The Fit Model



The Fit Model

```
#
# Call:
# glm(formula = cbind(Y, N - Y) ~ Dose, family = binomial, data = crypt
#
# Deviance Residuals:
#    Min      1Q  Median      3Q     Max
# -3.953  -1.244   0.233   1.553   3.601
#
# Coefficients:
#             Estimate Std. Error z value Pr(>|z|)
# (Intercept) -1.40777    0.14848   -9.48   <2e-16
# Dose         0.01347    0.00105   12.87   <2e-16
#
# (Dispersion parameter for binomial family taken to be 1)
#
#     Null deviance: 434.34  on 67  degrees of freedom
# Residual deviance: 200.51  on 66  degrees of freedom
# AIC: 327
#
```

The Meaning of a Logit Coefficient

Logit Coefficient: A 1 unit increase in a predictor = an increase of β increase in the log-odds ratio of the response.

$$Log\ Odds\ Ratio = Log\left(\frac{p_1}{1-p_1} \Big/ \frac{p_2}{1-p_2}\right)$$

We need to know both p1 and β to interpret this.

The Meaning of a Logit Coefficient

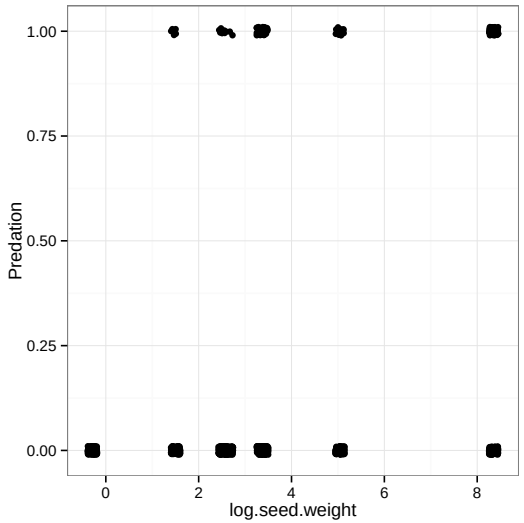
$$Log\ Odds\ Ratio = Log\left(\frac{p_1}{1-p_1} \Big/ \frac{p_2}{1-p_2}\right)$$

If p1 = 0.5, Odds Ratio = 1

With β = 0.01347, we multiply 1 by $e^{0.01347}$.

The new odds ratio is 1.013561, which means p=0.5033674

What if we Only Have 1's and 0's?



Seed Predators

http://denimandtweed.com

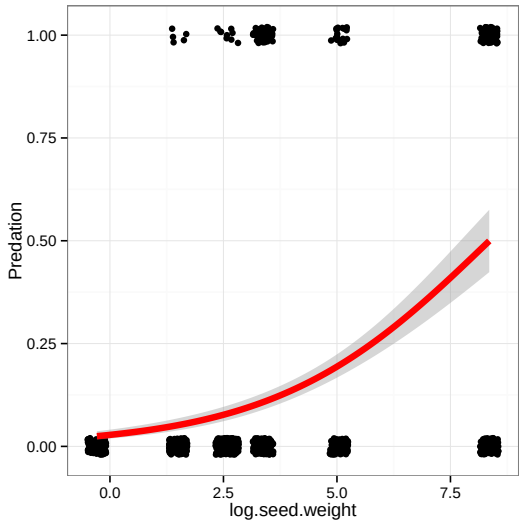
A Quick Note on Within and Transformation

```
seeds <- within(seeds, {
  log.seed.weight <- log(seed.weight)
})
```

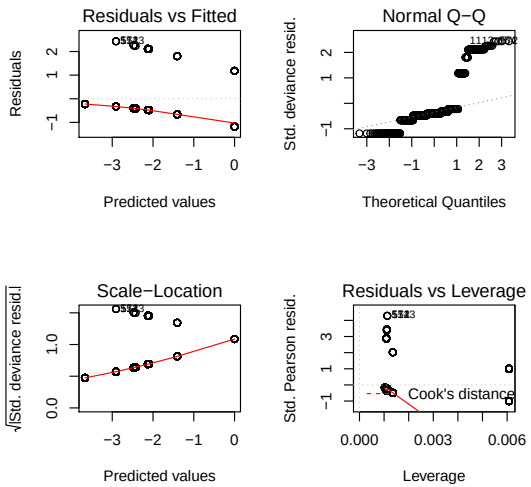
The GLM

```
seed.glm <- glm(Predation ~ log.seed.weight,
                data=seeds, family=binomial)
```

Fitted Seed Predation Plot



Diagnostics Look Odd Due to Binned Nature of the Data



Binned Residuals Should Look Spread Out